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Positive Kinematics Analysis of 6-3 Stewart Platform Parallel Manipulator

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ABSTRACT: *Aim at the question of direct kinematics solution of the 6-3 Stewart manipulator, this paper proposes a new type of rapid numerical solution to this parallel mechanism, this method can work out an accurate as well as unique solution., which is mainly uses the kinematics reverse solution features of the 6-3 Stewart manipulator, then get a linear equation involves the length microvariations of the rod and the position microvariations of the mobile platform. With adding the continuous micro variable which can get the positive kinematics solution of the 6-3 Stewart manipulator. Finally, take the reverse solution for the 6-3 Stewart manipulator for known condition, the direct kinematics solution is verified by a calculation example. At the same time, the Mathematica software is used to improve the calculate efficiency of the platform position.*

Keywords: *parallel mechanism; direct kinematics; numerical solution*

I. INTRODUCTION

In 1965, the British engineer D. steward proposed a kind of parallel mechanism which applied in flight simulator in his paper “A platform with Six Degree of Freedom” and it is namely Stewart manipulator[1]. The proposing of Steward manipulator is the opening of the study of parallel mechanism. Since then, due to these features (high rigidity, high load ability, high dynamic performance, high precision, compact structure, small movement inertia, reverse solution solved easily and so on), the parallel mechanism has became the hot spots at home or abroad[2].

Stewart platform parallel mechanism is composed of two platform and it has many configurations, one typical of them is called 6-3 type Steward platform, the static platform is hexagon and the mobile platform is triangle, O. Ma and j. Angeles once do a research of kinematic performance with different configurations of parallel mechanism, it comes out that the kinematic performance of parallel mechanism can achieve the best when the mobile platform is triangle[3].

To the question of direct kinematics solution, it is defined that the length of actuate rods are known and to solve the relative position and posture between mobile platform and static platform, scholars at home and abroad do a lot of research[4] to the forward kinematics of Stewart parallel mechanism[5], the basic method to solve this problem is numerical method and analytical method[6]. Literature [7] express positive kinematics solution problem with a one element eight equation, this equation is the latest achievements of analytic method. In literature [8], [9],a positive method for Steward manipulator proposed based on genetic algorithm, this algorithm make use of the global optimization characteristics of genetic algorithm, and overcome the disadvantage of direct selection.

A new type of fast numerical algorithm which mainly uses some kinematics reverse solution features of 6-3 Stewart manipulator proposed in this paper. This method can deduce a linear equation involves the length microvariations of the rod and the position microvariations of the mobile platform, and then get the positive kinematics solution and the positive kinematics solution is closest to the given initial value. At last, the numerical calculation and symbolic calculations is done by the software Mathematica.

II. SUMMARY OF THE BASIC THORY

According to the Taylor series expansion

$$f(x) - f(a) = (x - a)f'(x)$$

Get a multidimensional value approximation of Taylor first class seris

$$\|x + \delta\| - \|x\| \approx \delta \bullet b^T / \|b\| \tag{1}$$

Lie group and lie algebra oftern be used in robot kinematics and control. Here is the related concepts and symbols of lie group and lie algebra.

Special orthogonal group $SO(n)$: a subset of the general linear group

$$SO(n) = \{R \in GL(n, R) : RR^T = R^T R = I, \det R = \pm 1\} \tag{2}$$

In $SO(n) \in n(n-1)/2$, when n = 3, $SO(n)$ is called rotation group of R^3.

Special continental group $SE(3)$: the rigid body transformation group of R^3 is defined as a form of $g(x) = Rx + P$. Any elements in $SE(3)$ recorded as $(P, R) \in SE(3)$. $SE(3)$ can expressed by the following 4×4 matrix space:

$$g(P, R) = \begin{pmatrix} R & P \\ 0^T & 1 \end{pmatrix} \tag{3}$$

Where $R \in SO(3), P \in R^3$. $SE(3)$ *is a six-dimensional lie group.*

III. NUMERICAL ALGORITHM OF POSITIVE KINEMATICS SOLUTION

Figure 1 is a general 6-3 Stewart Manipulator structure, coordinate system fixed on the static and dynamic platform, P is position vector in static coordinate system, W is position vector in dynamic coordinate system. a_i is position vector of the dynamic platform vertex, b_i is position vector of the static platform vertex, l_i is the length of the rod.

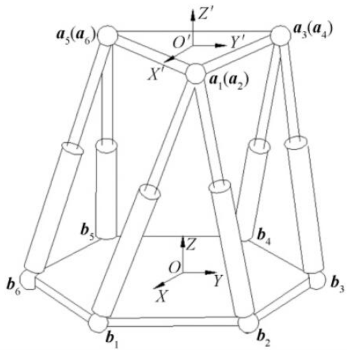

Fig.1 6-3 Stewart manipulator structure

The length of the rods can be defined by the rigid body transformation between static and dynamic platform:

$$l_i = \|g \bullet a_i - b_i\| \tag{4}$$

Where $\|\bullet\|$ denotes Euclidean norm of the vector and $a_i, b_i \in IR^3$. It can deduce the result

$$g \bullet a_i = R a_i + P \tag{5}$$

Equation (4) is kinematics reverse solution, it can be simplified to:

$$l_i = l_i(g) \tag{6}$$

Assumptions there is a subtle variation τ in equation (4), the dynamic platform also has subtle variation with respect to static platform, it can be described by the following equation:

$$l_i + \tau_i = \|g \bullet \gamma \bullet a_i - b_i\| \tag{7}$$

Where $\gamma = \begin{pmatrix} I_3 + \Omega & v \\ 0^T & 1 \end{pmatrix}$.

Relative pose between dynamic platform and static platform can be expressed by:

$$\varepsilon = (\gamma - I_4)^{\vee} = \begin{pmatrix} w \\ v \end{pmatrix} \tag{8}$$

Where $\vee$ operation transforms 4×4 screw matrix space into 6×1 vector, Ω is antisymmetric matrix, if use equation (7) minus equation (4), a new equation is obtained:

$$\tau_i = \|g \bullet \gamma \bullet a_i - b_i\| - \|g \bullet a_i - b_i\| \tag{9}$$

Through simplification

$$\tau_i = \|g \bullet \gamma \bullet a_i - b_i\| - \|g \bullet a_i - b_i\| = \|\gamma \bullet a_i - g^{-1} \bullet b_i\| - \|a_i - g^{-1} \bullet b_i\| \tag{10}$$

Based on equation (1) , it can be written as

$$\tau_i = \left\| a_i - g^{-1} \bullet b_i + \gamma \bullet a_i - a_i \right\| - \left\| a_i - g^{-1} \bullet b_i \right\|$$
$$= \frac{\left(a_i - g^{-1} \bullet b_i\right)^{\mathrm{T}} \left(a_i \times , - I_3\right)}{\left\| g^{-1} \bullet b_i - a_i \right\|} \bullet \varepsilon_i \tag{11}$$

Through simplification:

$$\tau_i = m_i^T \varepsilon_i \ \frac{\left(a_i - g^{-1} \bullet b_i\right)^{\mathrm{T}} \left(a_i \times , - I_3\right)}{\left\| g^{-1} \bullet b_i - a_i \right\|} \bullet \varepsilon_i \tag{12}$$

Where $a_i \times$ is a 3×3 antisymmetric matrix about a_i , $\varepsilon_i = \left(\gamma - I_4\right)^{\vee} = \begin{pmatrix} W \\ V \end{pmatrix}$. Substitute i=1,2,3,4,5,6 into the equation, threr will be six equations and these six equations can constitute a linear system of equations

$$\tau = M \varepsilon \tag{13}$$

Where τ is a given micro variable matrix, M can be obtained according to the initial transformation of the rigid body. ε can be obtained by this equation，and γ can be obtained by equation (8), then the rigid body transformation matrix of the platform will be $\mathbf{g} \bullet \gamma$. $\mathbf{g} \bullet \gamma$ will be respected as a new rigid body transformation matrix $\mathbf{g}$ and substituted into equation (13), a new ε can be obtained. Circulating above-mentioned process, through the iteration of $\mathbf{g} \bullet \gamma$ and track the movement trajectory of the rod, once all the length of the rods are close to the expectation length, iteration is completed, the result of direct kinematics solution will be obtained.

IV. PROVING THROUGH A NUMERICAL EXAMPLE

Using the kinematics inverse solution of the equation (4) with given initial position, initial length will be obtained, then calculate a set of inverse solution, that is, according to the position that the mobile platform have moved for a time t ,getting all the length of the rods in t time. Then test the accuracy of the numerical method: according to all the length of the rods in t time, the position will be found by the positive numerical solution, and comparing the position with expected position, the correctness and accuracy of the method are verified. xi

Static platform and dynamic platform of each vertex coordinates are shown in table 1, setting the circumradius of static platform is 2, and the circumradius of dynamic platform is 1, due to the platform is plane arrangement, all their Z component is zero.

Table 1 the parameters of the 6-3 Steward dynamic and static platform

i	bx_i	by_i	ax_i	ay_i
1	2cos(π/3)	2sin(π/3)	cos(π/6)	sin(π/6)
2	2cos(2π/3)	2sin(2π/3)	cos(π/6)	sin(π/6)
3	2cos(3π/3)	2sin(3π/3)	cos(5π/6)	sin(5π/6)
4	2cos(4π/3)	2sin(4π/3)	cos(5π/6)	sin(5π/6)
5	2cos(5π/3)	2sin(5π/3)	cos(9π/6)	sin(9π/6)
6	2cos(6π/3)	2sin(6π/3)	cos(9π/6)	sin(9π/6)

The initial position of the dynamic platform is

$$\mathbf{g_0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

As shown in figure 2, according to the equation (4) the length of the six rods is obtained, where $l_0 = [2.35285 \quad 2.35285 \quad 2.35285 \quad 2.35285 \quad 2.35285 \quad 2.35285]$

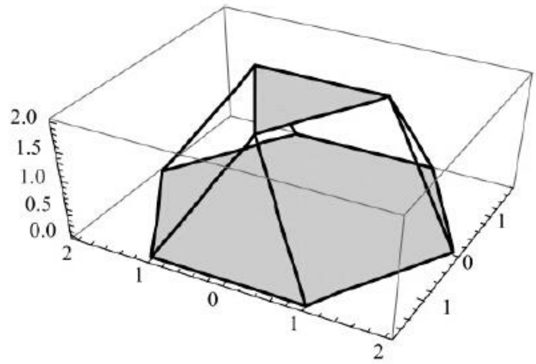

Fig.2 The initial position of the platform

According to the z-y-x euler Angle formula, set three euler angles as (π/6 π/4 π/3), the expected position is calculated as

$$R = R_z R_y R_x = \begin{pmatrix} 0.35355 & -0.57322 & 0.73920 \\ 0.61237 & 0.73920 & 0.28033 \\ -0.70711 & 0.35355 & 0.61237 \end{pmatrix} \tag{14}$$

$\boldsymbol{P} = [2\ 3\ 1]^T$ is a given position vector as shown in figure 3, so the expected position is calculated as

$$\mathbf{g_e} = \begin{pmatrix} 0.35355 & -0.57322 & 0.73920 & 2 \\ 0.61237 & 0.73920 & 0.28033 & 3 \\ -0.70711 & 0.35355 & 0.61237 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Combined with inverse solution formula (4), the length of the rods are caculated as

$l = [3.94061 \quad 2.45341 \quad 3.19765 \quad 4.78923 \quad 5.39856 \quad 4.34000]$

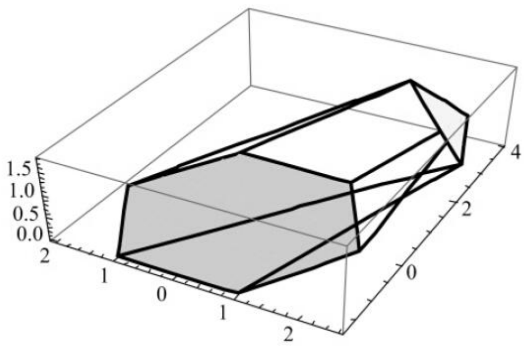

Fig.3 The expected position of the platform

Set the starting position of the platform and the length of the rods when the time is t as known conditions, assuming that each time the micro variable of the rods is 0. 01, if the initial length of the rods is shorter than the finally length, then $\tau_i = 0.01cm$, if the initial length of the rods is longer than the finally length, then $\tau_i = -0.01cm$.Using the kinematics Positive solution mentioned above as well as Mathematica, the finally position is obtained

$$\mathbf{g} = \begin{pmatrix} 0.37181 & -0.58479 & 0.73343 & 1.98830 \\ 0.60037 & 0.75247 & 0.29237 & 3.00250 \\ -0.72172 & 0.32403 & 0.62164 & 1.03430 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In this situation, the accuracy of the position is associated with micro variable of the rods .Such as, when $\tau_i = 0.001cm$, the finally position will be

$$\mathbf{g}' = \begin{pmatrix} 0.35566 & -0.57461 & 0.7384 & 1.99861 \\ 0.61098 & 0.74072 & 0.28172 & 3.00029 \\ -0.7087 & 0.35015 & 0.61358 & 1.00414 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

By comparison, it occurs that the results are nearly the same with expected position. Thus, parameter τ_i influence the speedability and accuracy of positive solution.

Position error and posture error is the maximum elements in $\mathbf{g_e}$, and the position error is bigger than the posture error, figure 4 is the relationship that position error and posture error influenced by different length variable τ_i obtained from Mathematica.

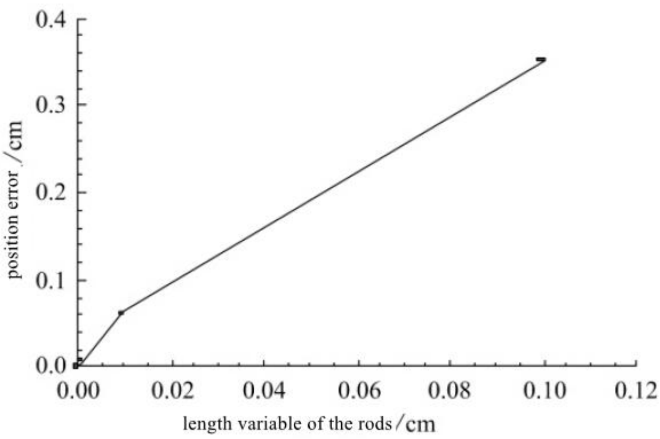

Fig.4 Position error changing with the length variable of the rods τ_i

If parameter τ_i is constant. When tracing the length variable path of the rods, in this process, to achieve the expected position and posture, the minimum number of iterations is

$$n_{\min} = \left|L_i - l_i\right|_{\max} / \tau_i \tag{15}$$

where L_i is finally length of the rods, l_i is initial length of the rods, i = 1,2,3,4,5,6. Apparently, when the biggest length variation can meet the conditions, the rest rods can reach the expected length, and the minimum number of superposition is the best. Figure 5 is the relationship that number of superposition influenced by different length variable τ_i obtained from Mathematica.

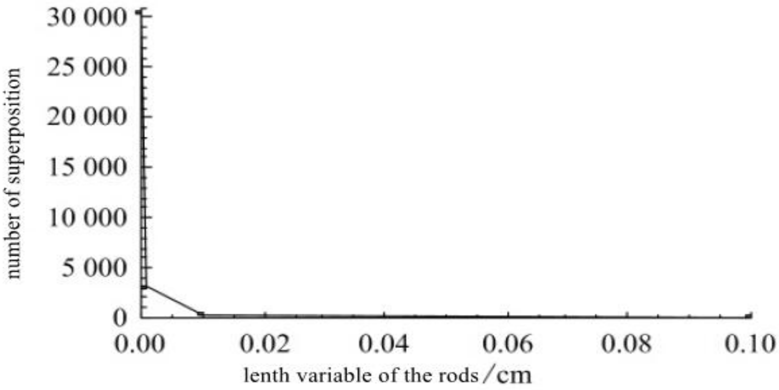

Fig.5 number of superposition changing with the length variable of the rods τ_i

V. CONCLUSION

This paper uses Mathematica to analyze the Positive Kinematics of 6-3 Stewart Platform Parallel Manipulator the 6-3 Stewart platform parallel mechanism kinematics positive solutions were analyzed, and puts forward a new rapid numerical solution method of type, can get the following conclusion.

With the analysis method of Lie group and lie algebra, and fully using the features that kinematics reverse solution of Stewart platform is relatively easy to be solved, the linear equations between the micro variable of the rods and platform are obtained, then the posture will be identified by the micro variable of the platform. The positive kinematics solution is obtained by tracking the length variable path of the rods until every rod get to the expected length value. Based on Mathematica symbolic computation software, program for calculating the Stewart parallel mechanism kinematics positive solution is obtained.

When accuracy requirement is more important than real-time requirement, τ_i values can be small as far as possible. On the contrary, when real-time requirement is more important than accuracy requirement, τ_i values can be big as far as possible. But once the value of τ_i is determined, the minimum number of iterations is determined. So, it can select suitable τ_i values according to different applications, this is the advantage with respect to other methods. However, this method has not overcome the deficiency of the initial value selection, and speedability and accuracy can not meet at the same time.

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